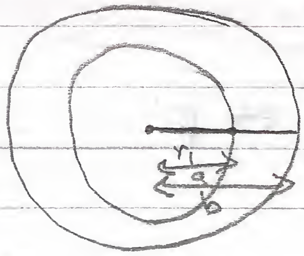
Sec 4

sheet 3

DATE 14-3-16

(9)

$\bar{F} = 5r^3 \bar{r}$



$\int_c \bar{F} \cdot d\bar{r} = \int_a^b 5r^3 r \, dr$

$= \int_a^b 5r^4 \, dr = r^5 \Big|_a^b = b^5 - a^5$

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sheet 3 ans

$\boxed{1}$ $\frac{1}{3}$ $\boxed{2}$ $\frac{1}{2}$ $\boxed{3}$ 1 $\boxed{4}$ 96π $\boxed{5}$ 236

$\boxed{6}$ • $-8/3$ • $\frac{44}{3}$ • 2π $\boxed{7}$ $\varphi = 3x^2y^2 - xy^2$

$\boxed{8}$ 5 $\boxed{9}$ $b^5 - a^5$

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sheet 4

* divergence Theory :-

$$\oint\!\!\oint_s \bar{F} \, d\bar{s} = \iiint_V \bar{\nabla} \cdot \bar{F} \, dv$$

* stokes Law :-

$$\oint_c \bar{F} \cdot d\bar{r} = \iint_s (\bar{\nabla} \times \bar{F}) \, ds$$

(3) $\bar{A} = ax\vec{i} + by\vec{j} + cz\vec{k}$

$$\oiint \bar{A} \cdot d\bar{s} = (a+b+c)V$$

According to ~~dig~~ divergance Th:

$$L.H.S = \oiint_S \bar{A} \cdot d\bar{s} = \iiint_V (\bar{\nabla} \cdot \bar{H})\, dv$$

$$= \iiint_V (a+b+c)\, dv = (a+b+c) \iiint_V dv$$

$$= (a+b+c)\, V = R.H.S$$

$$\bar{\nabla} \cdot \bar{A} = a+b+c$$

(5) $\iiint_V \frac{dv}{r^2} = \oiint_S \frac{\bar{r}\,\hat{n}\, ds}{r^2}$, $\hat{n}\, ds \to = d\bar{s}$

$\bar{\nabla} \bar{F} = \frac{1}{r^2}$ $\qquad \bar{F} = \frac{\bar{r}}{r^2}$

$$\bar{\nabla} \bar{F} = \bar{\nabla} \cdot \left(\frac{\bar{r}}{r^2}\right) = \frac{1}{r^2} (\bar{\nabla} \cdot \bar{r}) + \bar{r} \cdot \left(\bar{\nabla} \frac{1}{r^2}\right)$$

$$= \frac{3}{r^2} + \bar{r} \cdot \left(\frac{-2}{r^3}\right) \hat{r} \to \boxed{\frac{\bar{r}}{r}}$$

$$= \frac{3}{r^2} - \frac{2}{r^2} = \frac{1}{r^2}$$

$$R.H.S = \oiint_S \frac{\bar{r}}{r^2}\, d\bar{s} = \oiint_S \bar{F} \cdot d\bar{s}$$

$$= \iiint_V \bar{\nabla} \cdot \bar{F}\, dv = \iiint_V \frac{1}{r^2}\, dv = L.H.S$$

$\boxed{7}$ $\oint \frac{\partial u}{\partial x} dy - \frac{\partial u}{\partial y} dx = 0$ where C is a closed curve in The region R, 1

prove That is $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

$F_1 = \frac{\partial u}{\partial x}$ $\qquad F_3 = 0$

$F_2 = \frac{-\partial u}{\partial y}$

$$\bar{\nabla} \times \bar{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial u}{\partial x} & \frac{-\partial u}{\partial y} & 0 \end{vmatrix}$$

zero

$$= \left(\cancel{\frac{\partial \left(-\frac{\partial u}{\partial y}\right)}{\partial z}} - 0\right) - \left(\cancel{\frac{\partial \frac{\partial u}{\partial x}}{\partial z}} - 0\right)$$

$$= \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = R.H.S$$

$\boxed{9}$ Evalute $\oiint (y^2 z \vec{i} + x^3 \vec{j} + x\dot{z} \vec{k}) \cdot d\bar{s}$

$-1 \le x \le 1$, $-1 \le y \le 1$ and $0 \le z \le 2$

Sol.

$$\oiint_S \bar{F} \cdot ds = \iiint_V \bar{\nabla} \cdot \bar{F} \, dV$$

$$\bar{\nabla} \cdot \bar{F} = 0 + 0 + x = x$$

$$\int_0^2 \int_{-1}^1 \int_{-1}^1 x \, dx \, dy \, dz = \int_0^2 \int_{-1}^1 \frac{x^2}{2} \Big|_{-1}^{1} dy \, dz$$

$$= 0$$